Discussion 1

Monday, September 3, 2018 7:42 AM

Topics : Divide & Conquer , Complex number review

Divide & Conquer

- Algorithm :
  break up problem into smaller parts
  recursively solve smaller subproblems
  combine small solutions together
- Runtime :
  write runtime $T(n)$ in terms a recurrence relation
  $T(n) = aT(\frac{n}{b}) + O(\cdot)$ or $\theta(\cdot)$ or $\Omega(\cdot)$
  then $\underbrace{\text{solve the recurrence}}_{\text{refer to Discussion 0}}$ to get a bound
- Examples : exercise : recall the algorithm for each of below

→ multiplying 2 numbers :

$T(n) = 3T(n/2) + O(n) \Rightarrow T(n) = O(n^{\log_2 3}) \approx O(n^{1.59})$

→ merge sort :

$T(n) = 2T(n/2) + O(n) \Rightarrow T(n) = O(n \log n)$

→ finding k^{th} smallest (a randomized algorithm) : can be used to find median

best case : $T(n) = T(n/2) + O(n) \Rightarrow T(n) = O(n)$

worst case : $T(n) = T(n-1) + O(n) \Rightarrow T(n) = O(n^2)$

average case ?

after two iterations, array size will shrink to $\frac{3}{4}n$.

$T(n) = T(\frac{3}{4}n) + O(n) \Rightarrow T(n) = O(n)$

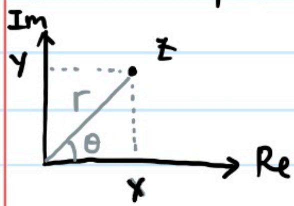
→ multiplying two matrices :

$T(n) = 7T(n/2) + O(n^2) \Rightarrow T(n) = O(n^{\log_2 7}) \approx O(n^{2.81})$

Complex numbers

- $z \in \mathbb{C}$ means

$z = x + iy$, where $x, y \in \mathbb{R}$



$Re(z) = x$

$Im(z) = y$

- idea : we can also specify z using angle and distance ...

polar coordinate :

Suppose $z = x + iy$.

$\Longleftrightarrow$ $z = re^{i\theta}$

- convertion between two coordinates

$$z = x + iy \underset{x = r\cos\theta,\ y = r\sin\theta}{\overset{r = \sqrt{x^2+y^2},\ \theta = \arctan\frac{y}{x}}{\rightleftarrows}} z = re^{i\theta}$$

- why polar coordinates ?

multiplication is easy !

Suppose $z_1 = r_1 e^{i\theta_1}$. $z_2 = r_2 e^{i\theta_2}$.

Then $z_1 \cdot z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

- why multiplying two complex numbers ?

we want to find n^{th} root of unity !

Suppose $n \in \mathbb{Z}^+$. Want to find $z \in \mathbb{C}$ such that

$$z^n = 1.$$

idea : in polar coordinates, $z^n = (re^{i\theta})^n = e^{i \cdot 0}$

$$\Rightarrow r^n e^{in\theta} = e^{i0}$$

$$\Rightarrow \begin{cases} r^n = 1 \\ n\theta = 0 + 2k\pi \text{ for } k \in \mathbb{Z}^+ \end{cases}$$

$$\Rightarrow \begin{cases} r = 1 \\ \theta = \frac{2k\pi}{n} \text{ for } \underbrace{k \in \{0, \dots, n-1\}}_{\text{e.g. } \theta = 0 \text{ and } \theta = 2\pi \text{ give the same point}} \end{cases}$$

Thus, $z = e^{i\frac{2k\pi}{n}}$ where $k \in \{0, \dots, n-1\}$

Okay. Why do we care n^{th} root of unity then ?

... too many questions (JK, you should ask these questions !!!)

It's used in FFT to efficiently multiply two polynomials