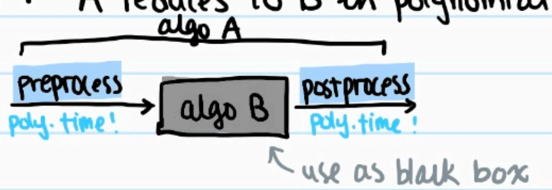
Discussion 10: NP-completeness

Sunday, November 4, 2018 8:03 PM

Polynomial Time Reduction

- $A \le_p B$: A reduces to B in polynomial time means



Sanity Check :

Q: In order to show $A \le_p B$, why do we use B as a black box?

A: We want to show

"there's no poly. algo. for A $\Rightarrow$ there's no poly. algo. for B".

Take the contraposition of the implication above :

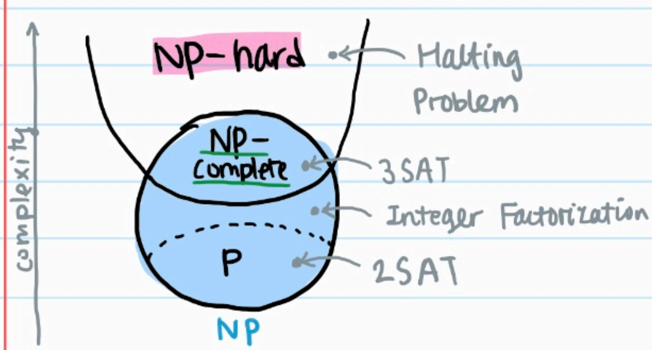
"$\exists$ poly algo. for B $\Rightarrow$ $\exists$ poly algo for A"

i.e. you want to use B as a black box and find polynomial time reduction (changing instance of A into instance of B).

Terminologies

- Search Problem
  - given an instance, find a solution
  - a solution can be verified in poly. time
- NP = { search problems }
- P = $\{ X \in NP \mid X$ can be solved in poly. time $\}$
- NP-complete = $\{ X \in NP \mid \forall Y \in NP,\ Y \le_p X \}$
- NP-hard = $\{$problem $H \mid \forall Y \in NP,\ Y \le_p H \}$

doesn't have to be in NP



Methods

- To show X is NP-hard :
find a NP-complete problem and reduce it to X
Side note : generally, if you want to show "for all ...", a single existance isn't enough. However, NP-complete problems all reduce to each other ; so showing one reduces to X also shows all reduce to X !
- To show X is NP-complete :
first show it's NP-hard ,
then show it's in NP.