Discussion 5

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Topic: More Greedy Algorithms

Huffman Encoding

- assign more frequent symbol shorter codeword by keeping joining the two nodes with the lowest frequencies.
- runtime : $O(n\log n)$

Horn Formula

Given a list of implications / pure negation, returns a satisfying assignment, if one exists.

- - set everything false
  - for every unsatisfied implications, set RHS to true "$\Rightarrow x$" ≡ "x is true"
  - check if all pure negations are still all satisfied
- runtime : n is the length of the formula
  - $O(n^2)$ naively
  - $O(n)$ cleverly

Set Cover

Given $S_1, \ldots, S_m \subseteq B$, select a selection of S_i's such that $\bigcup S_i = B$.

In addition, we want to minimize number of sets picked.

- Greedy (but not optimal) solution:
  pick the set with the largest number of uncovered elements
- Approximation factor : $\ln n$

Suppose optimal solution uses k subsets.

Let n_t be the number of uncovered elements after t iterations of greedy algorithm. (e.g. $n_0 = n$).

n_t are covered by k sets $\Rightarrow \exists$ some sets w/ at least $\frac{n_t}{k}$ uncovered elements

Thus,

$$n_{t+1} \leq n_t - \frac{n_t}{k} = n_t\left(1-\frac{1}{k}\right) \leq n_t e^{-\frac{1}{k}}$$

$$\Rightarrow n_t \leq n_0\left(1-\frac{1}{k}\right)^t < n_0\left(e^{-1/k}\right)^t = ne^{-t/k}$$

Notice that t is the # subsets in greedy solution, so we want to find what t is when $n_t < 1$.

$$1 = ne^{-t/k} \Rightarrow t = k\ln n$$

- runtime : $O(n)$