1. Counting II

a. pick k from n

	with replacement	without replacement
order matters	n^k	$\frac{n!}{(n-k)!}$, or nPk
order doesn't matter	$\binom{n+k-1}{n-1}$ or $\binom{n+k-1}{k}$	$\frac{n!}{(n-k)!k!}$, or $\binom{n}{k}$

b. distribute k balls among n bins

	without exclusion	with exclusion (maximum one ball in one bin)
distinguishable balls	n^k	$\frac{n!}{(n-k)!}$, or nPk
indistinguishable balls	$\binom{n+k-1}{n-1}$ or $\binom{n+k-1}{k}$	$\frac{n!}{(n-k)!k!}$, or $\binom{n}{k}$

*As you can see, even though we use different models, the formulae are the same! (Try to see why those two seemingly-different-scenario actually give you the same 'formula table'.)
Takeaway: You can use different models / approaches to solve the same problem! That's fun part of counting and probability. Don't forget to try to understand why they give you the same answer. (Doing this will help you see the larger picture of the problems).

2. Discrete Probability

a. Sample space / Outcome space (Ω): The set of all possible outcomes ω.

b. **Event**: A subset of sample space.

c. **Probability of an event for a uniform sample space**: $P(A) = \frac{|A|}{|\Omega|}$

3. Conditional Probability

a. **Conditional Probability**: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

b. **Total Probability Rule**: $P(A) = P(A|B)P(B) + P(A|\neg B)P(\neg B)$
(How do you generalize this to partition of Ω by more than two events?)

c. **Independence of two events**: $P(A \cap B) = P(A) \cdot P(B)$

d. **Mutual independence of more than two events**:
Events A_i's are mutually independent if $P(\cap_{i \in I} A_i) = \prod_{i \in I} P(A_i)$, $\forall I \subseteq \{1, ..., n\}$.

e. **Intersection of events (Product Rule)**:
$P(A \cap B) = P(A) \times P(B|A)$
(How do you generalize it?)

f. **Union of events (Inclusion/Exclusion Principle)**
Refer to the notes for formula because it's painful to Latex it, BUT more importantly, you should know:
Union Bound $P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$. Equal for disjoint events (why?).