1. Binomial Distribution

a. **Usage**: calculate the probability of having k success out of n independent trials, given the probability of success for each trial is p.

b. **Parameters**: $X \sim$ Binomial (n, p)

c. **PMF:** $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

2. Poisson Distribution

a. **Usage**: calculate the probability of having k events in a fixed time period, given the events occur with a known rate λ.

b. **Parameters:** $X \sim$ Poisson(λ).

c. **PMF:** $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$

3. Geometric Distribution

a. **Usage:** calculate the probability of having k independent trials to get the first success, given the probability of success for each trial is p.

b. **Parameters:** $X \sim$ Geometric(p)

c. **PMF:** $P(X = k) = (1-p)^{k-1} p$.

d. Application: The Coupon Collectors Problem
Scenario: n different coupons. infinitely many cereal boxes. each box constitutes one of the n coupons. Uniformly distributed at random. X is the random variable for number of boxes we need to buy in order to collect all n coupons.
Goal: calculate $E(X)$.
Solution:
$X = \sum X_i$
X_i : number of boxes I bought in order to collect the ith coupon, given that I already have coupon 1, 2, ..., i - 1.
$X_i \sim$ Geometric($\frac{n-(i-1)}{n}$)
$E(X_i) = \frac{n}{n-(i-1)}$
$E(X) = E(X_1) + \cdots + E(X_n) = \sum_{i=1}^{n} \frac{n}{n-(i-1)} = n(\frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{1}) = n \sum_{k=1}^{n} \frac{1}{k}$