

# DIS 4A

Sunday, July 8, 2018

9:29 PM

## Topics: Self-Reference and Computability

### The halting problem

- ```
def TestHalt(P, x):  
    if P(x) halts:  
        return True  
    else: P(x) loops forever  
        return False
```

```
def Turing(P):  
    if TestHalt(P, P) == True:  
        loop forever  
    else:  
        halt
```

Turing(Turing):

Case 1: halts  $\Rightarrow$  TestHalt(Turing, Turing) returns "no"  
 $\Rightarrow$  Turing(Turing) doesn't halt

Case 2: loops  $\Rightarrow$  TestHalt(Turing, Turing) returns "yes"  
 $\Rightarrow$  Turing(Turing) halts

programs are bit strings with finite length. They're countable and can be passed in as an argument.

|       | $P_1$ | $P_2$ | $P_3$ |         |
|-------|-------|-------|-------|---------|
| $P_1$ | H     | H     | L     |         |
| $P_2$ | L     | H     | H     |         |
| $P_3$ | H     | L     | L     |         |
|       |       |       |       | $\dots$ |

$\square \leftarrow \text{Turing(Turing)}$

So Turing does not exist on the list

$\Rightarrow$  TestHalt does not exist

Remark: the problem is about TestHalt. We use Turing to show TestHalt is not computable.

- Reduction: If you can use B to solve A...

$A \rightarrow B$ . A reduces to B. B is harder.

- Uncomputability proof:

Show P is not computable by using P to solve TestHalt.